import matplotlib.pyplot as plt

import math

# point cloud points sampled at equal intervals

# starts at wiki gate 24.5 and ordered by rising elevation

points29 = [[10000.434275, 10003.29959798, 151.95306396], [10000.64927298, 10003.26380706, 151.95298767],

[10000.75600702, 10003.24098802, 151.95304871], [10000.86140001, 10003.21367192, 151.95315552],

[10001.06930399, 10003.1503799, 151.95367432], [10001.27327597, 10003.07401991, 151.95375061],

[10001.47204101, 10002.98405004, 151.95346069], [10001.664433, 10002.8812561, 151.95310974],

[10001.848665, 10002.765872, 151.95291138], [10002.02572894, 10002.63908505, 151.95323181],

[10002.19383097, 10002.50103498, 151.95365906], [10002.35267401, 10002.35215402, 151.95321655],

[10002.50166702, 10002.19291401, 151.95341492], [10002.63981795, 10002.02498198, 151.95361328],

[10002.7668519, 10001.848261, 151.95370483], [10002.88197207, 10001.66335404, 151.95321655],

[10002.98499489, 10001.47148097, 151.95300293], [10003.07565904, 10001.27337205, 151.95321655],

[10003.15204597, 10001.06920898, 151.95359802], [10003.21514606, 10000.86077899, 151.95320129],

[10003.26531196, 10000.64801103, 151.95373535], [10003.30106902, 10000.43339199, 151.95327759],

[10003.30198908, 9999.56404099, 151.95335388], [10003.07742906, 9998.72447205, 151.95336914],

[10002.6429379, 9997.9708519, 151.95361328], [10002.02757597, 9997.35611391, 151.9538269],

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[9999.564852, 9996.69915104, 151.95318604], [9998.72571802, 9996.92403793, 151.95298767],

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[9998.72560203, 10003.07527804, 151.95361328], [9999.56531301, 10003.30008698, 151.9535675]]

points32 = [[10000.433972, 10003.29969406, 151.87738037], [10000.649872, 10003.26380992, 151.87692261],

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points35 = [[10000.43419901, 10003.30047989, 151.80081177], [10000.649077, 10003.26478791, 151.80123901],

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[10002.19444799, 10002.50208092, 151.80073547], [10002.35405302, 10002.35312009, 151.8006897],

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points38 = [[10000.433842, 10003.29881191, 151.7247467], [10000.648911, 10003.26341391, 151.7247467],

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points41 = [[10000.433402, 10003.295959, 151.64862061], [10000.64808601, 10003.26055408, 151.64810181],

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points47 = [[10000.43167499, 10003.28433108, 151.49023438], [10000.64532697, 10003.2489841, 151.48971558],

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[9996.72745705, 10000.430024, 151.35038757], [9996.95082688, 10001.26157999, 151.35003662],

[9997.38227391, 10002.00708389, 151.35002136], [9997.99157691, 10002.61506796, 151.35009766],

[9998.737849, 10003.0449841, 151.34999084], [9999.56965899, 10003.267946, 151.34991455]]

points56 = [[10000.42773399, 10003.25609112, 151.26713562], [10000.639983, 10003.22100592, 151.26750183],

[10000.74504799, 10003.19830298, 151.2671814], [10000.84908998, 10003.17217398, 151.26739502],

[10001.05562305, 10003.10918307, 151.2673645], [10001.25570405, 10003.03365898, 151.26734924],

[10001.45164096, 10002.94468594, 151.26759338], [10001.64101899, 10002.84327197, 151.2674408],

[10001.82385004, 10002.729568, 151.26792908], [10001.99795604, 10002.60436296, 151.26725769],

[10002.16414404, 10002.46798897, 151.26771545], [10002.32082391, 10002.32086897, 151.26754761],

[10002.46765995, 10002.16408396, 151.26739502], [10002.60413194, 10001.99771595, 151.26803589],

[10002.72948694, 10001.82317698, 151.26742554], [10002.84294009, 10001.64062297, 151.26737976],

[10002.94431496, 10001.45117104, 151.26721191], [10003.03372407, 10001.25480604, 151.26782227],

[10003.10925603, 10001.05446196, 151.2671051], [10003.17199898, 10000.848405, 151.26713562],

[10003.22104192, 10000.63947302, 151.26724243], [10003.25655699, 10000.42787099, 151.26724243],

[10003.25844789, 9999.569675, 151.26808167], [10003.03727889, 9998.74013102, 151.26783752],

[10002.608392, 9997.99648309, 151.26754761], [10002.00094509, 9997.38943911, 151.26808167],

[10001.25668705, 9996.96189809, 151.26716614], [10000.42749101, 9996.74115705, 151.26765442],

[9999.56999299, 9996.74223304, 151.2677002], [9998.74140203, 9996.96413302, 151.26757813],

[9997.99983692, 9997.39429688, 151.26727295], [9997.39460111, 9998.00118697, 151.26748657],

[9996.96445203, 9998.74244297, 151.26742554], [9996.74061608, 9999.57167599, 151.26715088],

[9996.73947597, 10000.42842001, 151.26808167], [9996.96231508, 10001.25706697, 151.26791382],

[9997.39201593, 10001.99937999, 151.2673645], [9997.99934101, 10002.60508609, 151.26742554],

[9998.74201596, 10003.0331459, 151.26724243], [9999.570187, 10003.2554791, 151.26730347]]

el29 = [0.229, 0.226, 0.187, 0.197, 0.197, 0.212, 0.199, 0.187, 0.19, 0.169, 0.177, 0.167, 0.187, 0.177, 0.197, 0.215,

0.222, 0.222, 0.227, 0.241, 0.257, 0.268, 0.326, 0.361, 0.397, 0.407, 0.387, 0.355, 0.329, 0.339, 0.305, 0.287,

0.325, 0.36, 0.387, 0.362, 0.338, 0.307, 0.285, 0.249]

el32 = [0.164, 0.152, 0.124, 0.205, 0.162, 0.178, 0.155, 0.162, 0.172, 0.142, 0.135, 0.143, 0.143, 0.157, 0.172, 0.192,

0.186, 0.188, 0.16, 0.174, 0.192, 0.207, 0.265, 0.305, 0.335, 0.35, 0.327, 0.295, 0.27, 0.327, 0.311, 0.232,

0.272, 0.31, 0.332, 0.309, 0.282, 0.244, 0.213, 0.205]

el35 = [0.169, 0.157, 0.119, 0.178, 0.169, 0.169, 0.163, 0.179, 0.151, 0.131, 0.159, 0.146, 0.142, 0.157, 0.174, 0.181,

0.179, 0.187, 0.139, 0.154, 0.174, 0.189, 0.249, 0.299, 0.328, 0.339, 0.311, 0.287, 0.26, 0.329, 0.314, 0.229,

0.269, 0.317, 0.329, 0.307, 0.27, 0.229, 0.199, 0.189]

el38 = [0.186, 0.197, 0.151, 0.201, 0.201, 0.19, 0.208, 0.186, 0.175, 0.156, 0.156, 0.166, 0.176, 0.166, 0.171, 0.186,

0.204, 0.203, 0.166, 0.186, 0.205, 0.216, 0.281, 0.326, 0.362, 0.376, 0.348, 0.316, 0.298, 0.354, 0.348, 0.256,

0.301, 0.348, 0.364, 0.336, 0.298, 0.258, 0.231, 0.22]

el41 = [0.162, 0.145, 0.136, 0.194, 0.205, 0.177, 0.161, 0.182, 0.162, 0.157, 0.138, 0.142, 0.152, 0.119, 0.133, 0.164,

0.162, 0.162, 0.142, 0.158, 0.174, 0.194, 0.262, 0.309, 0.358, 0.357, 0.328, 0.293, 0.279, 0.312, 0.337, 0.234,

0.278, 0.331, 0.347, 0.32, 0.281, 0.229, 0.199, 0.202]

el44 = [0.157, 0.148, 0.146, 0.148, 0.162, 0.132, 0.143, 0.125, 0.152, 0.119, 0.125, 0.107, 0.127, 0.117, 0.102, 0.145,

0.127, 0.137, 0.141, 0.156, 0.174, 0.197, 0.26, 0.312, 0.352, 0.359, 0.33, 0.297, 0.283, 0.308, 0.317, 0.24,

0.277, 0.337, 0.35, 0.322, 0.277, 0.228, 0.173, 0.157]

el47 = [0.203, 0.196, 0.192, 0.188, 0.176, 0.17, 0.162, 0.152, 0.146, 0.139, 0.138, 0.131, 0.131, 0.132, 0.139, 0.151,

0.157, 0.169, 0.181, 0.196, 0.212, 0.234, 0.304, 0.361, 0.399, 0.409, 0.376, 0.339, 0.326, 0.361, 0.321, 0.273,

0.321, 0.379, 0.401, 0.366, 0.321, 0.27, 0.215, 0.202]

el50 = [0.218, 0.209, 0.204, 0.201, 0.194, 0.184, 0.176, 0.167, 0.16, 0.166, 0.149, 0.14, 0.142, 0.144, 0.146, 0.154,

0.169, 0.174, 0.188, 0.204, 0.222, 0.244, 0.318, 0.374, 0.418, 0.424, 0.39, 0.351, 0.334, 0.352, 0.317, 0.28,

0.334, 0.394, 0.414, 0.383, 0.334, 0.279, 0.234, 0.214]

el52 = [0.240, 0.231, 0.227, 0.222, 0.212, 0.204, 0.195, 0.188, 0.177, 0.168, 0.159, 0.156, 0.158, 0.158, 0.161, 0.168,

0.177, 0.194, 0.202, 0.219, 0.235, 0.259, 0.336, 0.393, 0.435, 0.442, 0.408, 0.372, 0.352, 0.373, 0.328, 0.292,

0.354, 0.415, 0.437, 0.402, 0.35, 0.292, 0.239, 0.235]

el56 = [0.259, 0.251, 0.244, 0.24, 0.231, 0.222, 0.212, 0.204, 0.192, 0.179, 0.173, 0.17, 0.17, 0.17, 0.171, 0.178,

0.187, 0.198, 0.211, 0.237, 0.248, 0.272, 0.351, 0.41, 0.453, 0.461, 0.427, 0.384, 0.371, 0.384, 0.342, 0.301,

0.37, 0.433, 0.457, 0.419, 0.364, 0.303, 0.252, 0.251]

new = []

for p in el29:

new += [p \* .0254]

el29 = list(new)

new = []

for p in el32:

new += [p \* .0254]

el32 = list(new)

new = []

for p in el35:

new += [p \* .0254]

el35 = list(new)

new = []

for p in el38:

new += [p \* .0254]

el38 = list(new)

new = []

for p in el41:

new += [p \* .0254]

el41 = list(new)

new = []

for p in el44:

new += [p \* .0254]

el44 = list(new)

new = []

for p in el47:

new += [p \* .0254]

el47 = list(new)

new = []

for p in el50:

new += [p \* .0254]

el50 = list(new)

new = []

for p in el52:

new += [p \* .0254]

el52 = list(new)

new = []

for p in el56:

new += [p \* .0254]

el56 = list(new)

new = []

points = points29 + points32 + points35 + points38 + points41 + points44 + points47 + points50 + points52 + points56

dist = el29 + el32 + el35 + el38 + el41 + el44 + el47 + el50 + el52 + el56

# starting parameters

xc = 10000.000615397612

yc = 10000.00040379931

zc = 151.82316465831497

rc = 3.3240036718929082

alpha = 0.0007 # learning rate

mse = 0

while mse > 0.0004:

x\_change = 0

y\_change = 0

z\_change = 0

r\_change = 0

mse = 0

for i in range(0, len(points)):

mse += (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \*\* 2

x\_change += 2 \* (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \* (

(1 / 2) \* (1 / math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)))) \* (

2 \* (xc - points[i][0]))

y\_change += 2 \* (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \* (

(1 / 2) \* (1 / math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)))) \* (

2 \* (yc - points[i][1]))

z\_change += 2 \* (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \* (

-0.5 \* (1 / (math.sqrt(rc \*\* 2 - (zc - points[i][2]))))) \* (-2 \* (zc - points[i][2]))

r\_change += 2 \* (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \* (

-0.5 \* (1 / (math.sqrt(rc \*\* 2 - (zc - points[i][2]))))) \* (2 \* rc)

mse = math.sqrt(mse / len(points))

print(mse)

rmse = (1 / mse) \* 0.5

x\_change = (x\_change / len(points)) \* rmse

y\_change = (y\_change / len(points)) \* rmse

z\_change = (z\_change / len(points)) \* rmse

r\_change = (r\_change / len(points)) \* rmse

xc = xc - alpha \* x\_change

yc = yc - alpha \* y\_change

zc = zc - alpha \* z\_change

rc = rc - alpha \* r\_change

print(xc, yc, zc, rc)

# xc = 10000.000615212366

# yc = 10000.000403627007

# zc = 151.82313134652927

# rc = 3.3244697186758834

xc = 10000.000639526275

yc = 10000.00042639083

zc = 151.82326315456376

rc = 3.3239622768323454

# xc = 10000.0007

# yc = 10000.0005

# zc = 151.826

# rc = 3.324

for i in range(0, len(points)):

mse += (math.sqrt(((xc - points[i][0]) \*\* 2) + ((yc - points[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points[i][2]) \*\* 2)) - dist[i]) \*\* 2

mse = math.sqrt(mse / len(points))

print(mse)

y = [1.0, 1.25, 1.375, 1.5, 1.75, 2.0, 2.25, 2.5, 2.75, 3.0, 3.25, 3.5, 3.75, 4.0, 4.25, 4.5, 4.75, 5.0, 5.25, 5.5,

5.75, 6.0, 7.0, 8.0, 9.0, 10.0, 11.0, 12.0, 13.0, 14.0, 15.0, 16.0, 17.0, 18.0, 19.0, 20.0, 21.0, 22.0, 23.0, 24.0]

x29 = []

for i in range(0, len(points29)):

x29 += [1 \* (math.sqrt(((xc - points29[i][0]) \*\* 2) + ((yc - points29[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points29[i][2]) \*\* 2)))]

f29 = plt.figure()

plt.title("29\" elevation")

plt.plot(y, x29, label="point cloud")

plt.plot(y, el29, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("29(resample).png")

plt.close()

x32 = []

for i in range(0, len(points29)):

x32 += [1 \* (math.sqrt(((xc - points32[i][0]) \*\* 2) + ((yc - points32[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points32[i][2]) \*\* 2)))]

f32 = plt.figure()

plt.title("32\" elevation")

plt.plot(y, x32, label="point cloud")

plt.plot(y, el32, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("32(resample).png")

plt.close()

x35 = []

for i in range(0, len(points29)):

x35 += [1 \* (math.sqrt(((xc - points35[i][0]) \*\* 2) + ((yc - points35[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points35[i][2]) \*\* 2)))]

f35 = plt.figure()

plt.title("35\" elevation")

plt.plot(y, x35, label="point cloud")

plt.plot(y, el35, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("35(resample).png")

plt.close()

x38 = []

for i in range(0, len(points29)):

x38 += [1 \* (math.sqrt(((xc - points38[i][0]) \*\* 2) + ((yc - points38[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points38[i][2]) \*\* 2)))]

f38 = plt.figure()

plt.title("38\" elevation")

plt.plot(y, x38, label="point cloud")

plt.plot(y, el38, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("38(resample).png")

plt.close()

x41 = []

for i in range(0, len(points29)):

x41 += [1 \* (math.sqrt(((xc - points41[i][0]) \*\* 2) + ((yc - points41[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points41[i][2]) \*\* 2)))]

f41 = plt.figure()

plt.title("41\" elevation")

plt.plot(y, x41, label="point cloud")

plt.plot(y, el41, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("41(resample).png")

plt.close()

x44 = []

for i in range(0, len(points29)):

x44 += [1 \* (math.sqrt(((xc - points44[i][0]) \*\* 2) + ((yc - points44[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points44[i][2]) \*\* 2)))]

f44 = plt.figure()

plt.title("44\" elevation")

plt.plot(y, x44, label="point cloud")

plt.plot(y, el44, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("44(resample).png")

plt.close()

x47 = []

for i in range(0, len(points29)):

x47 += [1 \* (math.sqrt(((xc - points47[i][0]) \*\* 2) + ((yc - points47[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points47[i][2]) \*\* 2)))]

f47 = plt.figure()

plt.title("47\" elevation")

plt.plot(y, x47, label="point cloud")

plt.plot(y, el47, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("47(resample).png")

plt.close()

x50 = []

for i in range(0, len(points29)):

x50 += [1 \* (math.sqrt(((xc - points50[i][0]) \*\* 2) + ((yc - points50[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points50[i][2]) \*\* 2)))]

f50 = plt.figure()

plt.title("50\" elevation")

plt.plot(y, x50, label="point cloud")

plt.plot(y, el50, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("50(resample).png")

plt.close()

x52 = []

for i in range(0, len(points29)):

x52 += [1 \* (math.sqrt(((xc - points52[i][0]) \*\* 2) + ((yc - points52[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points52[i][2]) \*\* 2)))]

f52 = plt.figure()

plt.title("52\" elevation")

plt.plot(y, x52, label="point cloud")

plt.plot(y, el52, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("52(resample).png")

plt.close()

x56 = []

for i in range(0, len(points29)):

x56 += [1 \* (math.sqrt(((xc - points56[i][0]) \*\* 2) + ((yc - points56[i][1]) \*\* 2)) - math.sqrt(

(rc \*\* 2) - ((zc - points56[i][2]) \*\* 2)))]

f56 = plt.figure()

plt.title("56\" elevation")

plt.plot(y, x56, label="point cloud")

plt.plot(y, el56, color='green', label="physical")

plt.legend(loc='upper left')

plt.xlabel("Wicket Gate")

plt.ylabel("Distance (m)")

plt.savefig("56(resample).png")

plt.close()

"""

x = []

y = []

for point in points:

x += [point[0]]

y += [point[1]]

fig, ax = plt.subplots()

circle2 = plt.Circle((xc, yc), rc, color='b', fill=True)

ax.plot(x, y, 'o')

ax.add\_artist(circle2)

fig.savefig('plotcircles2.png')

"""